

Dr. George F. Spagna, Chmn.  
Department of Physics  
Randolph-Macon College  
Ashland, VA 23005-5505

March 3, 2007

Dear Dr. Spagna:

Thank you for your letter of Feb. 28. Also, I recall that you were helpful in my attempt to get aboard computers with my aerospace problem back in 1999. I still have the disc you gave me with our work with spread sheets. (I do not think that was the most productive approach as it turned out, but what was most helpful was your directing my attention toward the small calculator with a coding feature and a large memory. I found one soon after which has been most helpful in my work.)

It is good to hear that the R-MC Physics Department is thriving and growing. In my years there, I thought the courses under Drs. Wade Temple, Richard Grove, and Patrick Clay were interesting and challenging--at times almost too challenging. I will certainly keep May 8 in mind on my calendar.

You might be interested in seeing the results achieved less than a year ago of my time related trajectory project. I am enclosing a short synopsis of those results. Of course, I am not in the best position to evaluate my own work, but after working on it on and off for forty years, the results seem much better than I would ever have guessed possible.

My best wishes to Randolph-Macon, the Physics Department, and to you.

Sincerely,

*Wood Rosenberger*

Wood Rosenberger  
414 Central St. #15  
Elkins, WV 26241

## Introduction

It has been some months now since we have discovered that some arbitrary trajectory of any  $(r_1, \theta, r_2)$  configuration is related to some central angle  $\theta_c$  of a circular orbit. In other words, if we are able to find the  $\theta_c$  related to any trajectory in which we might be interested, then we can solve that trajectory and find all of its elements.

It has been only in recent weeks that we have been able to reduce the complications of our solution to its present form.

We still find it helpful in making the calculations more manageable to use the convention where  $(r_1)$  is (unit) scalar value and  $(\mathcal{M})$  is unit gravity constant. It follows that  $(\sqrt{\frac{\mathcal{M}}{r_1}})$  is unit velocity,  $(\sqrt{\mathcal{M} r_1})$  is unit angular momentum,  $(\sqrt{\frac{r_1^3}{\mathcal{M}}})$  is unit time, and  $(r_1^2)$  is unit area.

*Wood Rosenberger*

Wood Rosenberger

6-16-06

## Time Related Trajectories

Let us say that we are interested in trajectories of a configuration that has  $(r_1) = 2.2E7$  ft,  $r_2 = 2.0E7$  ft, and the central angle  $\Theta = 40$  deg. The gravity  $M = 1.4E16$  ft<sup>3</sup>/sec<sup>2</sup>. Then the angular momentum is in units of  $(\sqrt{\mu r_1}) = 5.54977477E11$  and the time is in units of  $(\sqrt{\frac{r_1^3}{\mu}}) = 872.1074639$  sec. The angle  $\beta_0$  is defined by

$$\cot \beta_0 \equiv \frac{\left(\frac{r_1}{r_2} - \cos \Theta\right)}{\sin \Theta}$$

and 
$$N_p \equiv \sqrt{\cot^2 \frac{\Theta}{2} + 2 \cot \beta_0 \cot \frac{\Theta}{2} - 1}$$

Here  $\beta_0 = 62.54620048$  and  $N_p = 3.066511925$

There exists a central angle of a circular orbit such that  $0 < \theta_c < 180$  deg. that is related to each trajectory in the configuration between the extremely shallow parabolic trajectory and the very lofty "a" trajectory (the latter which is characterized by a major axis  $2a = r_1 + r_2$  and an area above the chord that is equal to half the entire orbital area). We have two equations that relate the circular angle, one to the primary angle gamma, and one to the angle of elevation, beta, of the trajectory in question.

$$\cot \gamma_i = \cot \frac{\Theta}{2} + N_p (\sin \frac{\Theta}{2} \cot \gamma_c - \cos \frac{\Theta}{2}),$$

$$\text{where } \tan \gamma_c = \frac{\theta_c^r - \sin \theta_c}{1 - \cos \theta_c}$$

$$\text{and } \cot \beta_i = N_p \cos \frac{\Theta}{2} - \cot \frac{\Theta}{2}$$

with both beta and gamma, we can quickly find both the angular momentum  $h$ , and the time  $t$  as well as any other element of the trajectory that might interest us.

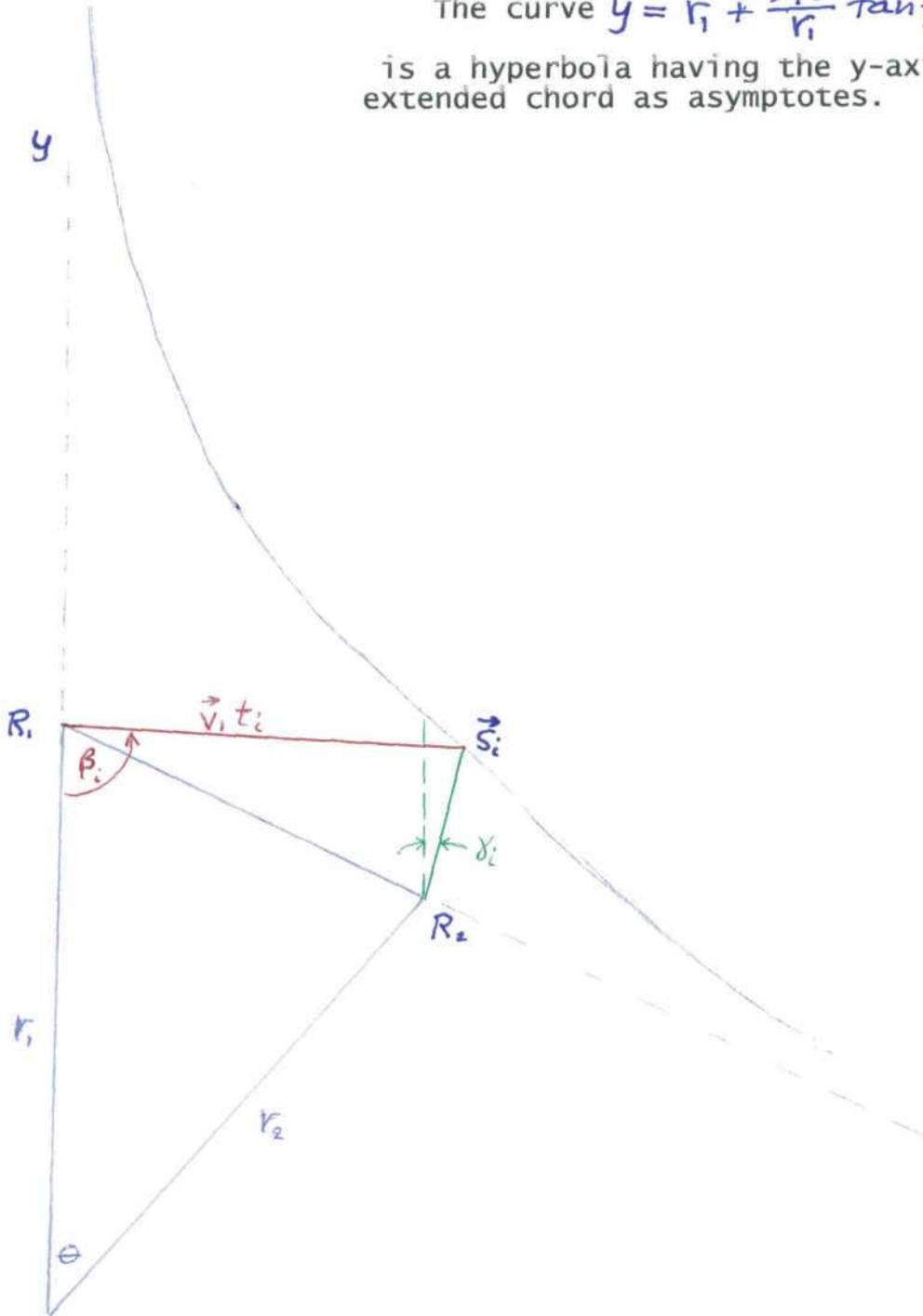
$\cot \beta_i = \cot \beta_0 - \frac{(4r_1)}{h_i^2} \tan \frac{\Theta}{2}$  gives us the angular momentum which in turn gives us the initial velocity  $\vec{v}_i = (\dot{x}, \dot{y})$

$$\text{where } \dot{x} = \frac{h}{r_i} \text{ and } \dot{y} = -\dot{x} \cot \beta_i$$

$$\text{we have } S_x = \frac{r_1 + r_2 (\sin \Theta \cot \gamma_i - \cos \Theta)}{(\cot \gamma_i + \cot \beta_i)}$$

$$t_i = \frac{S_x}{\dot{x}} = \frac{r_i S_x}{h_i}$$

The curve  $y = r_1 + \frac{4t^2}{r_1} \tan \frac{\theta}{2} \left( \frac{1}{x} \right) - x \cot \frac{\theta}{2}$   
 is a hyperbola having the y-axis and the  
 extended chord as asymptotes.



we can prove  $\vec{S}_i$  lies somewhere on the curve. If  $\gamma = \gamma(t_i)$   
 (that is, if we have the proper angle gamma for the time  $t_i$   
 ), then the problem is essentially solved.

### Example 1

Let us take the arbitrary case where  $\theta_c = 73.45 \text{ deg} = 1.281944336 \text{ radians}$ .

then  $\gamma_c = 24.33134371$   
and  $\gamma_i = 12.96120601$  for our "i" trajectory

for our second equation,

$$\beta_i = 106.152217$$

$$s_x = 0.7009506142(r_i)$$

$$h_i = 0.6706783712 = 3.722139E11 \text{ ft}^2/\text{sec}$$

$$\dot{x} = 0.6706783712 = 16918.69956 \text{ ft/sec}$$

$$\dot{y} = 0.1942436894 = 4900.039667 \text{ ft/sec}$$

$$t = s_x/\dot{x} = 1.04513675 = 911.4715622 \text{ sec.}$$

---

### Example 2

Let us now find the time for a trajectory for which we know the angular momentum  $h$ . For example, the minimum energy trajectory has  $h_i = h_m = (\sin \beta_0 \tan \frac{\theta_c}{2})^{1/2} (\sqrt{\mu r_i})$ . For our configuration, then  $h_m = .5683141326$

$$\beta_m = 121.2731002 \quad (\text{notice } \beta_m = 90 + \frac{\beta_0}{2})$$

$$\text{Then } \theta_c = 91.48288798$$

$$\gamma_c = 30.19737274$$

$$\text{and } \gamma_i = 12.85720674$$

$$s_x = 0.7688436469$$

$$t = 1.335253873 \left( \frac{\sqrt{r_i}}{\mu} \right) = 1164.484869 \text{ sec.}$$

### Example 3

The converse problem is much more difficult. That is, finding the angular momentum for a trajectory with a given time requires several attempts before we converge on the right answer. Fortunately, we have a good approximation of the angular momentum based on an estimate of the prime angle gamma. It is accurate enough that we are able to converge on the right answer in about four loops.

$$h_{ij} = \left(\frac{h_{oi}}{2}\right) + \sqrt{\left(\frac{h_{oi}}{2}\right)^2 + \lambda_j} \quad \text{where } h_{oi} = \frac{(r_1)r_2 \sin\theta}{t_i}$$

$$\lambda_j = \frac{(4r_1) \tan \frac{\theta}{2}}{\cot \beta_o + \cot \gamma_j}$$

When  $\gamma_j = \gamma_i$ , then  $h_{ij} = h_i$ . Let us find  $h_i$  for the trajectory in our configuration for which time  $t_i = 1500$  sec.  $= 1.71997152 \sqrt{\frac{r_1^3}{a}}$ .  $h_{oi}/2 = 0.169872689$

We start off with the estimate of  $\theta_e = 90$  (+10<sup>-12</sup>)

$\theta_e = 90$	110.0504573	110.3855998	110.3862145
$\gamma_c = 29.71756582$	36.15930066	36.26606406	36.26625984
$\gamma_j = 12.86663166$	12.72590377	12.72330729	12.72330252
$h_{ij} = 0.4910805481$	0.4899076476	0.4898859526	0.4898859134
$\beta_{:i} = 134.7035371$	134.9122026	134.9160621	134.9160692
$\theta_e = 110.05045727$	110.38009	110.3862033	110.3862145
$\Delta\theta_e = 20.05045727$	0.3296327766	6.03463283E-4	1.8438E-8

So with  $\theta_e$  (input) =  $\theta_e$  (output), then  $\gamma_j = \gamma_i = 12.72330252$  and  $h_{ij} = h_i = 0.4898859134$

We achieved convergence in just four loops by extrapolating  $\Delta\theta_e$  versus  $\theta_e$  which is very helpful if we are using a simple calculator with no coding feature. If we have the benefit of a computer, it is simpler to simply feed the output value back into the loop. Computers do not mind two or three more loops.