Physics 152

Spring 2008

Written homework problem #4 (due 2008 March 10)

A horizontal concave mirror with radius of curvature r = 50 cm holds a layer of water with maximum depth d = 1 cm. At what height above the surface of the water must you place an object in order that the image is at the same position as the object?

Recall that the way to "eat an elephant" is "one bite at a time."

First, let's find the properties of the optical components in the system: The water acts as a *plano-convex lens*. Its focal length may be found from the "lens maker's formula," with $n_{water} = 4/3$:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

The first surface is planar, so $r_1 = \infty$, $r_2 = 50\text{cm}$. The focal length of the lens is thus

$$f_{lens} = \frac{r_2}{n-1} = \frac{50cm}{1/3} = 150cm$$

The focal length of the mirror is just

$$f_{mirror} = r_2/2 = 25\text{cm}$$

This is a "thin lens" so we will measure all distances from the same point, roughly the top surface of the water. The lens forms an image which serves as the object for the mirror. The mirror forms an image which is the object for another pass through the lens. The final image must be real. Let the object distance be *s*. We have

$$\frac{1}{s}+\frac{1}{s_1}=\frac{1}{f_{lens}}$$
$$\frac{1}{s_1}=\frac{1}{f_{lens}}-\frac{1}{s}$$

(We'll leave it in this form since it's easier to manipulate algebraically, rather than solving for s_1. BTW – note that we're leaving the numbers until the end!) The object distance for the mirror is negative (virtual object), equal to $-s_1$. The mirror forms an image at s_2.

$$\frac{1}{s_2}-\frac{1}{s_1}=\frac{1}{s_2}-\frac{1}{f_{lens}}+\frac{1}{s}=\frac{1}{f_{mirror}}$$
$$\frac{1}{s_2}=\frac{1}{f_{mirror}}+\frac{1}{f_{lens}}-\frac{1}{s}$$

We repeat the process, with the object distance for second pass through the lens set as $-s_2$. The final image distance is just s.

$$\frac{1}{s}-\frac{1}{s_2}=\frac{2}{s}-\frac{1}{f_{mirror}}-\frac{1}{f_{lens}}=\frac{1}{f_{lens}}$$
$$\frac{2}{s}=\frac{2}{f_{lens}}+\frac{1}{f_{mirror}}$$

$$\Rightarrow s=\frac{2f_{lens}f_{mirror}}{2f_{mirror}+f_{lens}}$$

Upon substitution, we find s = 37.5 cm.

By the way, there's a "quick and dirty" path through this problem. For thin lenses (and, by extension, mirrors in contact with thin lenses) we can show that the focal lengths add inversely:

$$\frac{1}{f_{equivalent}} = \frac{1}{f_{lens}} + \frac{1}{f_{mirror}} + \frac{1}{f_{lens}}$$

Substituting for the lens and mirror focal lengths, we find the equivalent focal length of the system is $f_{equivalent} = 18.75$ cm. We want object and image distances the same, so

$$\frac{2}{s} = \frac{1}{f_{equivalent}}$$
$$\Rightarrow s = 2f_{equivalent} = 37.5cm$$

[Most common mistake on this problem was to leave out the final pass through the lens, turning a 3-component system into 2 components.]