Finding the minimum semi-definite rank of a graph

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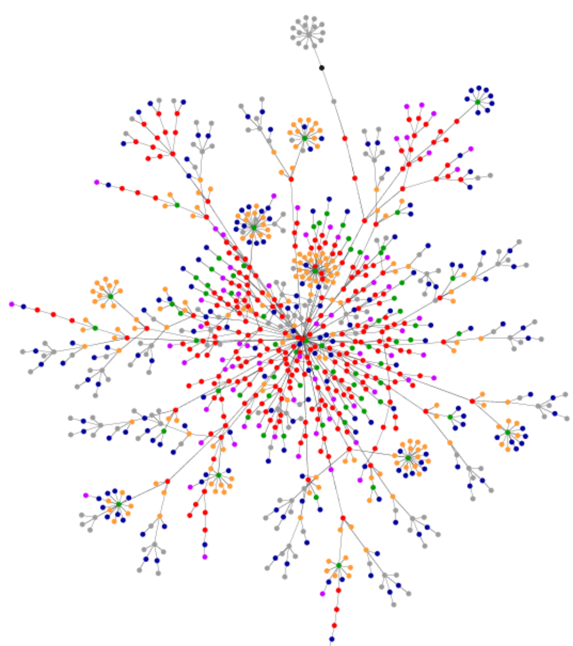
A matrix $A \in M_n(\mathbb{C})$ is called *Hermitian* if $A = A^*$. A Hermitian matrix with nonnegative eigenvalues is called a *positive semi-definite (PSD)* matrix. Given a Hermitian matrix A we associate a simple, undirected graph G with vertices $V(G) = \{1, \ldots, n\}$ and edges $E(G) = \{ (i,j) \mid a_{ij} \neq 0, i \neq j \}$. This associated graph is independent of the diagonal entries of A. Given a graph G, the *minimum semi-definite rank* of G, denoted $\operatorname{msr}(G)$, is the minimum of rank of A as A varies over all PSD matrices with graph G.

In this talk we present results on the upper and lower bounds for $\operatorname{msr}(G)$, and the effect of topological changes such as vertex or edge modifications on $\operatorname{msr}(G)$. In addition, we will discuss the $\operatorname{msr}(G)$ for some classes of graphs, including bipartite graphs and chordal graphs.

The talk should be accessible to any undergraduate student who has taken the first course in linear algebra.



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