

# Japanese Temple Geometry: Traditional Mathematics During the Tokugawa Period

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## Warm-up Problem

In Japanese mythology, the crane and the turtle are symbols of good fortune and longevity.

Consider the following problem, common to present day Japanese elementary education:

**Q:** There is a group of cranes and turtles, 50 in all. The total number of legs is 122. How many cranes and how many turtles are there?

**A:** 39 cranes and 11 turtles.

**D:** Now discuss with your students how they arrived at their answers. Is it possible to do this problem without solving a system of linear equations? (Hint: Japanese elementary students would say “yes”!)

## Historical Context

After the Battle of Sekigahara in 1600, Tokugawa Ieyasu rises to the shogunate.

All foreign missionaries are expelled from Japan, and Western presence becomes limited to a station of Dutch traders residing strictly on a small manmade island called Dejima.

Thus begins Japan’s period of *sakoku*, or “closed country” (c. 1640-1850). While Japan continues contact with its Asian neighbors, the flow of goods and information from Europe slows to a trickle.

**D:** What effects might you expect this major foreign policy change to have on Japanese society? Government? Technology and science? What would happen if the United States cut off all contact with Europe today?

Among other things, *sakoku* results in a renaissance of the arts called *Genroku*.

**Q:** When you think of Japanese art and culture, what specific forms come to mind?

It is likely that many of the responses to the question will have originated during the *Genroku* of the Tokugawa period. A few examples are below.

- *Ukiyo-e* (“floating world”) paintings and woodblock prints.
- Haiku poetry
- Kabuki Theatre
- Origami, Tea Ceremony, and Flower Arranging

In addition, a homegrown, grassroots form of mathematics called *wasan* arises in Japan.

*wasan* = *wa* “Japan” + *san* “mathematics”

Here is a picture of some *wasan* in action:

<http://www.physics.princeton.edu/~trothman/maththumb.jpg>

**D:** What do you see happening here? Does anything look familiar? What appears different or surprising? What can you infer about the way Japanese mathematics was being practiced during the Tokugawa period?

## History of Math

Mathematics developed slowly over a long period of time, and evolved at different paces in a variety of locations and eras. Some important mathematical milestones:

- c. 200 - 500 BC (Greece) Pythagoras, Euclid, Archimedes
- c. 100 (China) Decimal notation
- c. 800 (Persia) *Al-jabr*
- c. 900 (India) Notion of “zero” as a stand-alone number
- 1494 (Italy) Symbols for “plus” and “minus”
- 1637 (France) Descartes, symbolic geometry
- c. 1675 (England, Germany) Calculus invented

Notice that, before Descartes, mathematics lacked a symbolic language and was always written longhand. Once this language was developed, and Calculus was invented, mathematical innovation exploded.

**Q:** Why was Japan *not* a part of this mathematical revolution?

**A:** These major modern innovations occurred in Europe, right around the time *sakoku* began.

## Development of *Wasan*

By 1650, the stage had been set:

- Japanese isolation and cultural blossoming
- Chinese mathematical influence, including the abacus
- Basic geometry from the Greeks, including the Pythagorean theorem and trigonometry
- Relative peace and stability

**Q:** The last item in this list had an important effect on the samurai class. What might that be?

**A:** With no more wars to fight, samurai became bureaucrats, courtiers, patrons of the arts, and teachers in small private academies called *juku*.

**D:** Find a picture/image of the *juku* of Sakuma Yoken (1819-1896) in Tamura City, where mathematics was taught. Presenting this to students, could this be a *juku*?

Several editions of the *Jinko-ki*, a Japanese math textbook, were published between 1627 and 1829.

**D:** Find a picture/image of pages from the *Jinko-ki*. What do the problems in this textbook appear to be about? What does this say about the interests of the reader?

While much of the math in the *Jinko-ki* had practical application for merchants and farmers, the samurai preferred the beauty of geometry.

**D:** Around this time, what was the relationship between the samurai and merchant classes? How might this relationship affect the scholarship and interests of these two groups?

### Wasan Geometry

Here are some problems that were solved in *juku* Tokugawa Japan. You can find the answers at the end of this document.

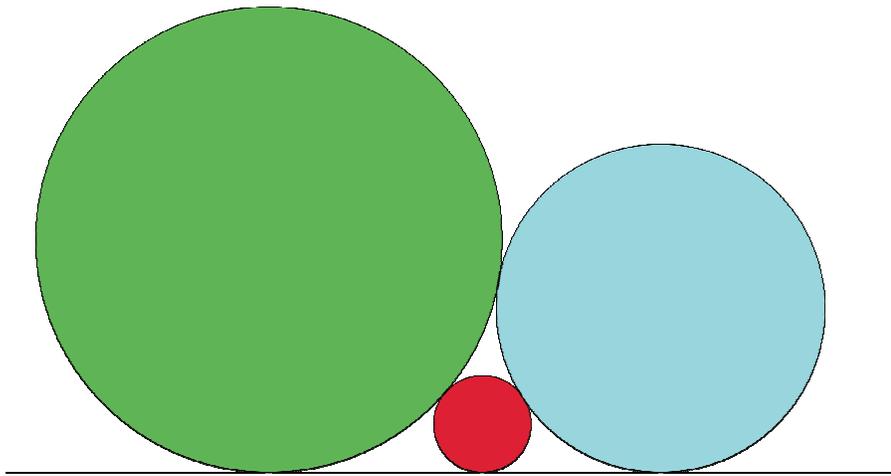
**Problem 1:** Look at the diagram on the front of this article:

<http://www.its.caltech.edu/~ilian/ma1c/temple.pdf>

Given that the radius of the green circle is 1, find the radius of the largest blue circle. (circa 1788)

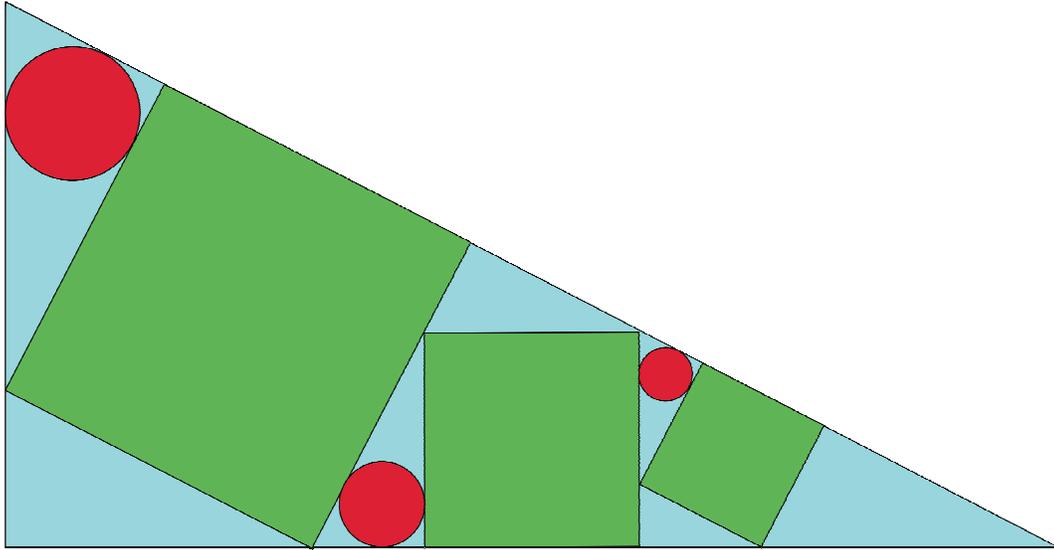
(Warning: the problem is *very* hard!)

**Problem 2:** Find an equation relating the radii of these three circles. (circa 1824)



(You should try this one! Hint: It requires only the Pythagorean theorem.)

**Problem 3:** Find an equation relating the radii of these three circles. (circa 1913)



(Hint: Use similar triangles.)

Huge numbers of juku schools, run by samurai, churned out solutions to countless problems like this. High volume and vast geographic spread made it impossible to compile printed lists of these problems. Instead, problem solvers needed another outlet for displaying their work. Take a look at the temple photographed below.

<http://www.physics.princeton.edu/~trothman/shrine.jpg>

Under the roof of this temple, wooden tablets have been hung. Zoom in for a closer look!

**Q:** What do you see on this tablet?

**A:** Geometric diagrams!

This is an example of a *sangaku*.

*sangaku* = *san* "math" + *gaku* "tablet"

*Wasan* practitioners in a *juku* would solve problems and paint them on wooden tablets – with answers, but no solutions. These *sangaku* would then be hung under the eaves of a nearby Shinto shrine or Buddhist temple as an offering to the gods, and as a challenge to other worshippers.

Here are some *sangaku* that survive today:

- Katayamahiko shrine, Murahisagun Okayama city, 1873
- Sugawara Tenman shrine, Mie prefecture, 1854
- Myoujyourinji temple, Ogaki city, 1865
- Mizuho shrine, Nagano prefecture, 1800
- Abe no Monjyuin temple, Fukushima prefecture, 1877
- Dewasanzan shrine, Yamagata prefecture, 1823
- Onnma shrine, Aichi prefecture, 1797

It is unknown how many *sangaku* were created and posted during the time that *wasan* flourished, but there are about 900 surviving tablets scattered across Japan.

Not all *sangaku* problems were geometric. The following problem came from a 1743 tablet found in the Kurasako Kannon temple:

“There are 50 chickens and rabbits. The total number of feet is 122. How many chickens and how many rabbits are there?”

Of course, this is the warm-up problem from the beginning of the lesson.

**D:** What does this say about the kinds of people who created *sangaku* problems?

**For further research/discussion:** Starting in the 1850's, with the arrival of Commodore Perry, Japan reopened to the West. At that time, Japanese higher mathematics had been largely focused on geometry problems and the solving of certain polynomial equations. Western mathematics, however, was far more advanced, and had spread to influence (and be influenced by) fields like physics and astronomy. What happened to *wasan* once Japan reestablished contact with Europe?

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Answers to the problems above:

Problem 1. The radius is  $1/[(2n-1)^2 + 14]$ .

Problem 2.  $1/\sqrt{r_3} = 1/\sqrt{r_1} + 1/\sqrt{r_2}$ , where  $r_3$  is the radius of the smallest circle.

Problem 3.  $r_2^2 = r_1 r_3$ , where  $r_2$  is the radius of the medium circle.